

# **DYNAMICS OF BUBBLE AND DROP IN MULTIPHASE FLOW THROUGH NUMERICAL SIMULATION**

Thesis submitted in partial fulfillment of the requirement for the degree of

**Master of Technology**

In  
**Mechanical Engineering**

By  
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Under the guidance of  
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## **CERTIFICATE**

*This is to certify that the thesis entitled “Dynamics of Bubble and Drop in Multiphase Flow through Numerical Simulation”, submitted by Gagandeep (Roll Number: 213ME3421) to National Institute of Technology, Rourkela, is a record of bona fide research work under my supervision, to the best of my knowledge in partial fulfilment of the requirements for the degree of Master of Technology in the Department of Mechanical Engineering, National Institute of Technology Rourkela.*

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- b. The work has not been submitted to any other Institute for any degree or diploma.
- c. I have followed the guidelines provided by the Institute in writing the thesis.
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<b>NOMENCLATURE</b>	
$Re$	Reynolds number
$u$	Magnitude of velocity, m/s
$P$	Pressure force, N/m <sup>2</sup>
$D_h$	Hydraulic diameter of the channel, mm
$C_p$	Specific heat at constant pressure, J/kg-K
$k$	Thermal conductivity, W/m-K
$D$	diameter

<b>Greek symbol</b>	
$\rho$	Density, kg/m <sup>3</sup>
$\mu$	Dynamic viscosity, N -s/m <sup>2</sup>
$\alpha$	Thermal diffusivity
$\nabla$	Differential operator

<b>Subscript</b>	
L	liquid
G	Gas

<b>Superscript</b>	
T	Transpose

## ABSTRACT

An attempt is made to study the hydrodynamics of drop and bubble under various boundary conditions (including adiabatic and non-adiabatic boundary condition) using numerical simulations. The effect of various parameters like: drop diameter, Reynolds no., position of drop on the dynamics are studied. The numerical analysis of the drop and bubble is done using Finite Volume Method (FVM) with Volume of Fluid (VOF) model. At first, the kinetics of a drop on a horizontal solid surface under air flow (considering no heat transfer) is studied. Then the same problem with heat transfer through solid surface is considered. At last the hydrodynamics of a helium bubble through a vertical channel is studied. For each case the Reynolds number is varied for a considerable range.

*Keywords—Drop; Bubble; Hydrodynamics; Helium bubble; Shear flow; Adiabatic Condition; Evaporation; Finite Volume Method (FVM; Volume of Fluid (VOF); Buoyancy.*

# **CHAPTER 1:**

# **INTRODUCTION AND**

# **LITERATURE SURVEY**

## 1.1 INTRODUCTION

The controlled and manipulated movement of liquid drops and bubbles have many applications in various fields. When a liquid drop is placed on a solid surface or a bubble is introduced in some fluid medium, its shape, size and motion depends on a number of factors such as fluid properties, the type and nature of surface and the surrounding atmosphere. The factors that affect the behavior of a drop or a bubble while their movement on a solid surface or through a fluid medium include material properties, the type of interaction of a drop or bubble with the solid surface or the fluid medium, surface tension, contact angle, gravity effect, surrounding medium etc. In this study effort has been done to study about the effect of some of these factors on the behavior of drop and bubble. The study of the behavior of bulk fluid is different from that of the study of dynamics of drop and bubble because in case of drop and bubble inertial and viscous forces are dominated by the surface tension force.

### 1.1.1 Principles

In this section, a brief description of the fundamental physics of drop has been given. Origin and introduction to surface tension has been covered in section 1.2.1. In section 1.2.2 the physics of drop in contact with solid surface is considered and a brief introduction to the types of surfaces on the basis of contact angle and Young's equation is given.

**1.1.1.1 Surface Tension:** Drops are considered to be the volumes of liquid which are bounded by some immiscible interface and they are associated with an interfacial tension. To understand the dynamics of drop, knowledge of surface or interfacial tension is must. Every molecule in a fluid experiences a mutual attraction due the other neighboring

molecules. All the molecules in the fluid have neighbors that attracts the molecules equally from all the directions except the molecules at the surface. The molecules at the surface have less number of neighbors and thus experiences net force. Now to overcome this net force they have to do some work. The work done by the molecules at the surface to overcome the net force is save in the form of interfacial tension. Surface tension is directly related to the intermolecular attraction and indirectly related to the size of the molecules. That is as the intermolecular interaction increases and the size of the molecules decreases, the surface tension also increases. Surface tension is expressed as force per unit length or energy per unit area. It can be considered equivalent to a negative surface pressure. Pressure act throughout the fluid whereas surface tension is a phenomena that is confined to the free surface only.

Boundary of the drop marks the change in the density, so it experiences a net gravitational force. If a bubble is placed in the water, it would rise and at the same time a raindrop falls in air. A drop moving in air experiences a resisting force which has hydrostatic, viscous and dynamic pressure components. The shape and the speed of the drop moving in another fluid depends upon the magnitude of these stress components.

**1.1.1.2 Wetting: Interaction With Solids:** Contact angle also has a great influence on the movement of a drop on a solid surface. When liquid is in contact with the solid surface, there exist an interface between the two. The angle made by the tangent to the liquid surface with the point of contact at the interface between the liquid and the solid surface is known as contact angle. Contact angle decides the wettability property of the liquid. If the contact angle is greater than  $90^\circ$ , the fluid does not wets the surface and is called as hydrophobic surface. If the contact angle is less than  $90^\circ$ , the fluid wets the

surface and is called as hydrophilic. At  $0^\circ$  the fluid completely wets the surface. Hydrophobic surfaces have low solid surface free energy and poor adhesiveness whereas hydrophilic surfaces have good adhesiveness and high solid surface free energy.

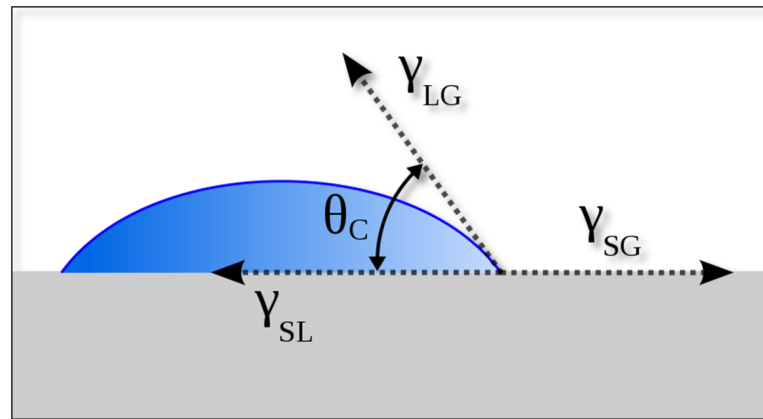


Fig 1.1 Contact angle of a liquid droplet wetted to a rigid solid surface (Source: [Wikipedia](#))

The contact angle  $\theta$ , the interfacial tension  $\gamma_{SL}$ , the surface tension of the liquid, the surface free energy of the solid are related to each other and the relationship between them is given by the Young's equation.

$$\gamma_{SG} = \gamma_{SL} + \gamma_{LG} \cos \theta \quad (1.1)$$

### 1.1.2 Application

Study of behavior of drop and bubble is gaining importance now a day because of its various engineering applications. Its use in microfluidic devices giving this topic an immense importance. It can be used for cooling of hot spots in micro scale applications. Manipulation of drop and bubble can be used in the design of seals and valve; it can also be used in macromixing, micro reaction and in micro pumping applications.



## 1.2 LITERATURE REVIEW

Study of drop and bubble dynamics is gaining immense importance all over the world now a days. Various analysis has been done to study the behavior of drop while it moves between parallel plates. To study the phenomena of droplet impact on a solid surface had been studied numerically first by [Harlow and Shannon \(1967\)](#). Then the analysis of the dynamics of molten droplet in thermal spray processes was done by [Tsurutani et al. \(1990\)](#) and [Trapaga and Szekeley \(1991\)](#). A 3D model had been developed by [Busmann et al. \(1999 and 2000\)](#) for the simulation of impacts of water droplets later on [Pasandideh-Fard et al. \(2002\)](#) modified the model to analyze the impact of any metal droplet. [Staben et al., \(2003\)](#) made the use of boundary- integral method to study the dynamics of spherical and ellipsoidal particles. They varied the size and location of the particle in the channel. For various particle sizes and particle location they calculated the rotational and translational velocities of the particles during their motion in the channel.

Study of dynamics of drop through cylindrical tubes is related to our study has gained lots of attention because of its various applications. Its various fields of application include biomechanics, the movement of blood cells through nerves, study of gas bubble in the blood streams etc. The study of movement of drop through straight tubes had been carried out by [Olbricht and Kung \(1992\)](#). [Martinez and Udell \(1990\)](#) studied about the behavior of deformable drop which was placed in pressure-driven flow. [Tsai and Miksis \(1994\)](#) performed their analysis on the dynamics of axisymmetric drop and bubble during their motion through a straight tube. They study the behavior of drop and bubble for various capillary number. The study of semi-infinite drop during its motion through a cylindrical tube was undertaken by [Hodges et al. \(2004\)](#).

Due the inherent property of mass conservation, VOF model is more preferred for the numerical analysis of drop spreading. The computational costs incurred by using this method is comparatively less and it is suitable for solving the problems involving large surface topology variation. Apart from various advantages associated with this model, it is also having some drawbacks such as the accuracy of interface calculations is less when compared to other methods such as front tracking , level set method etc. [Fukai et al., 1995](#); [Sikalo et al., 2005](#); [Gunjal et al., 2005](#); had clearly showed that despite of all the shortcomings of this model, it is the most widely used and most preferred method for numerical calculation of drop dynamics. How the spreading phenomena of drop is influenced by the wettability gradient is shown by [Fukai et al. \(1995\)](#). The impact velocity play a vital role in the spreading phenomena of droplet. [Pasandideh-Fard et al. \(2002\)](#) made the use of continuum surface force (CSF) model and interface tracking algorithm for three- dimensional solidification study of a molten drop. The impact of drop on horizontal surface has been studied experimentally and numerically by [Gunjal et al. \(2005\)](#). They used the VOF model for the numerical analysis of the drop dynamics for various wettability gradient. Most of the above studies conducted to study the dynamics of drop include horizontal surface.

There are many studies conducted by various researchers to study the behavior of drop over inclined surface and for various wettability gradient. The upward movement of a water drop has been demonstrated by [Chowdhury](#) and [Whitesides](#). Decyltrichlorosilane has been polished over the silicon wafer to obtain the desired velocity of the drop. Wettability influenced motion of liquid drop has been analyzed for the first time by [Huang et al.](#), using numerical method based on lattice Boltzmann technique. The

behavior of a liquid drop on the surface with surface energy gradient has been analyzed using finite element method by [Liao et al.](#) [Das et al.](#), used numerical scheme of diffused interface to study the dynamics of droplet over an inclined plane.

So many efforts have been done over the years to describe the shape, size, stability and movement of the drop over the surface. Sliding of drop over an inclined plane considering the “pouring of the liquid” has been undertaken by [Frenkel](#). He demonstrated that in case wetting of surface, a thin unstable film of liquid is being left behind by the liquid drop. [Brown et al.](#) used the finite element method to study about the shape of the drop placed on an inclined plane. He assumed that the contact line between the drop and the inclined plane as circular. One –dimensional periodic drop profiles has been studied by [Thiele et al.](#), for the drop sliding over an inclined plane using diffused interface theory and long wave approximation. They also extended their study for two-dimensional drops and studied the instabilities and movement of drops on an inclined plane.

The knowledge of how liquid drops behave when come in contact with the solid substrate is been utilized in spray cooling of surfaces. This has been studied by [Grissom et al.](#), (1981). They studied about the evaporation phenomena of droplet. [Attinger et al.](#), (2000) studied about the solder jetting phenomena. Eutectic solder has been utilized by them to study about the impact and solidification phenomena of a molten drop on a solid surface for the given time and length scale. Surface properties such as surface defects has a great influence on the movement of a drop on the non-ideal real substrates has been demonstrated by [Beltrame et al.](#),(2001). Some driving force is essential for overcoming the micro scale heterogeneities of the substrate and make the drop move. [Daniel](#) and

[Chaudhury \(2002\)](#) studied about the effect of wettability gradient on the movement of a liquid drop on a solid substrate. They demonstrated that the speed attained by the drop is about 1-2 mm/s if the drop moves due to the wettability gradient. Contact angle hysteresis which act against the driving force is responsible for such a low speed. With the help of a flexible glass micro needle, [Hitoshi](#) and [Satoshi \(2003\)](#) calculate the value of driving force generated as a result of contact angle gradient. [Wu et al. \(2003\)](#) studied about the effect of drop size and initial velocity of the drop on the spreading phenomena of the drop over the solid substrate. For the partial wetting, study about the shape and movement of the drop over an inclined plane has been carried by [Grand et al., \(2005\)](#). [Krasovitsky \(2005\)](#) studied about the measurement of contact angle for various polymeric materials. They also studied about the hysteresis behavior of dynamic and static contact angles.

The motion of a drop over a surface has been demonstrated by [Pismen](#) and [Thiele \(2006\)](#) using an asymptotic model which is based on the lubrication theory. Verification of such a model has been done by using tetra ethylene glycol drop placed on a silicon surface by [Subramanian et al., \(2006\)](#). Spreading phenomena of a drop over a solid substrate at high rate has been numerically analyzed by [Ding et al., \(2007\)](#) to understand the effect of inertia forces. [Hongwen et al., \(2010\)](#) studied about the effect of gravity on the shape and focal length changes of a drop. They conducted their study for various drop sizes and varying environment conditions. The contact line dynamics and the internal fluid structure has been studied by [Das](#) and [Das \(2010\)](#). The numerical analysis of the droplet has been done by using smoothed particle hydrodynamics (SPH).

### **1.3 GAPS IN LITERATURE SURVEY**

Though lots of study has been conducted related to drop and bubble dynamics but not much work has been done to analyze the dynamics of drop on flat plate under shear flow. Dynamics of Helium bubble placed in a rectangular tube filled with water is not much studied.

### **1.4 AIMS AND OBJECTIVE OF THE WORK**

The aim of this paper is to study about the dynamics of drop and bubble under shear flow with and without heat flow. The effect of parameters like Reynolds number, drop diameter and location of the drop on the kinetics of drop and bubble has been analyzed. This paper also includes the study of evaporation phenomenon of the drop on a horizontal solid surface under shear flow with non-adiabatic conditions. Analysis of dynamics of helium bubble is also carried out by introducing a helium bubble in a long rectangular tube which is filled with water. To analyze the shape change and motion of the drop and bubble in a multiphase flow, volume of fluid (VOF) with 3D Finite Volume Method (FVM) has been considered.

### **1.5 ORGANIZATION OF THESIS**

Chapter 2 describes the problems along with suitable assumptions and boundary conditions. The methodology used to solve the above said problems is described in chapter 3. This chapter also includes the information about the grid pattern employed, governing equations, boundary conditions and residual convergence. The results obtained along with required discussions are considered in chapter 4. Chapter 5 includes conclusion and future work.

# **CHAPTER 2:**

# **PROBLEM**

# **DESCRIPTION**

Dynamics of drop and bubble are studied here by considering several cases which are explain below:

## 2.1 THE DYNAMICS OF DROP

The study of drop dynamics has been studied numerically by considering three different types of problems. A rectangular parallelepiped of dimensions  $30\text{ mm} \times 10\text{ mm} \times 20\text{ mm}$  is taken. A hemispherical drop has been placed on the solid bottom surface of rectangular domain that is filled with air. The following boundary conditions have been given:

- Left plane of the domain as *velocity inlet*;
- Right side of the domain as *pressure outlet*;
- Top plane of the domain as *pressure inlet*;
- All other planes has been given the condition as *wall*;

For the better analysis of the problem, three different cases have been considered as shown below:

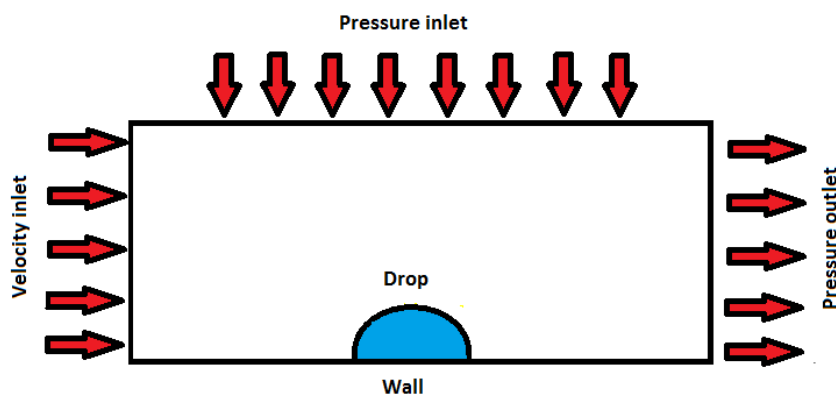


Figure 2.1: Pictorial representation of problem with all the boundary conditions.

**2.1.1** The effect of Reynolds No. on the dynamics of the drop on flat horizontal plate under shear flow with adiabatic condition by considering three different sub-cases with drop diameter equals to 6 mm

**2.1.1.1** In this section, the value of Reynolds number is taken as 1500, the drop is placed at  $X=15$  mm,  $Y=0$  mm,  $Z=10$  mm from the velocity inlet plane.

**2.1.1.2** In this section, the value of Reynolds number is taken as 3000, the drop is placed at  $X=15$  mm,  $Y=0$  mm,  $Z=10$  mm from the velocity inlet plane.

**2.1.1.3** In this section, the value of Reynolds number is taken as 4000, the drop is placed at  $X=15$  mm,  $Y=0$  mm,  $Z=10$  mm from the velocity inlet plane.

**2.1.2** The effect of location on the dynamics of the drop on flat horizontal plate under shear flow with adiabatic condition by considering different sub-cases.

**2.1.2.1** In this section, the diameter of the drop is taken as 6 mm. the drop is placed at  $X=5$  mm,  $Y=0$  mm,  $Z=10$  mm from the velocity inlet plane. The value of Reynolds number is taken as 3000.

**2.1.2.2** In this section, the diameter of the drop is taken as 6 mm. the drop is placed at  $X=15$  mm,  $Y=0$  mm,  $Z=10$  mm from the velocity inlet plane. The value of Reynolds number is taken as 3000.

**2.1.2.3** In this section, the diameter of the drop is taken as 10 mm. The drop is placed at  $X=5$  mm,  $Y=0$  mm,  $Z=10$  mm from the velocity inlet plane. The value of Reynolds number is taken as 3000.

**2.1.2.4** In this, the diameter of the drop is taken as 10 mm. The drop is placed at  $X=15$  mm,  $Y=0$  mm,  $Z=10$  mm from the velocity inlet plane. The value of Reynolds number is taken as 3000.



**2.1.3** The effect of size on the dynamics of the drop on flat horizontal plate under shear flow with adiabatic condition by considering different sub-cases

**2.1.3.1** In this section, the diameter of the drop is taken as 6 mm. the drop is placed at  $X=15$  mm,  $Y=0$  mm,  $Z=10$  mm from the velocity inlet plane. The value of Reynolds number is taken as 3000.

**2.1.3.2** In this, the diameter of the drop is taken as 10 mm. The drop is placed at  $X=15$  mm,  $Y=0$  mm,  $Z=10$  mm from the velocity inlet plane. The value of Reynolds number is taken as 3000.

**2.1.4** The effect of heat flux on the dynamics of the drop on flat horizontal plate under shear flow with non-adiabatic condition by considering three different sub-cases.

**2.1.4.1** In this case, heat flux equals to 1 MW is given to the bottom wall of the rectangular parallelepiped domain. Reynolds number is taken as 2000 to analyze how the drop evaporates while moving on the solid plate. The diameter of the drop is taken as 6 mm. The drop is located close to the velocity inlet plane i.e.  $X=5$  mm,  $Y=0$  mm,  $Z=15$  mm.

**2.1.4.2** In this case, heat flux equals to 10 MW is given to the bottom wall of the rectangular parallelepiped domain. Reynolds number is taken as 2000 to analyze how the drop evaporates while moving on the solid plate. The diameter of the drop is taken as 6 mm. The drop is located close to the velocity inlet plane i.e.  $X=5$  mm,  $Y=0$  mm,  $Z=15$  mm.

**2.1.4.3:** In this case, heat flux equals to 100 MW is given to the bottom wall of the rectangular parallelepiped domain. Reynolds number is taken as 2000 to analyze how the drop evaporates while moving on the solid plate. The diameter of the drop is taken as 6

mm. The drop is located close to the velocity inlet plane i.e.  $X=5$  mm,  $Y=0$  mm,  $Z=15$  mm.

## 2.2 HYDRODYNAMICS OF HELIUM BUBBLE THROUGH THE VERTICAL CHANNEL FILLED WITH WATER CONSIDERING ADIABATIC WALL

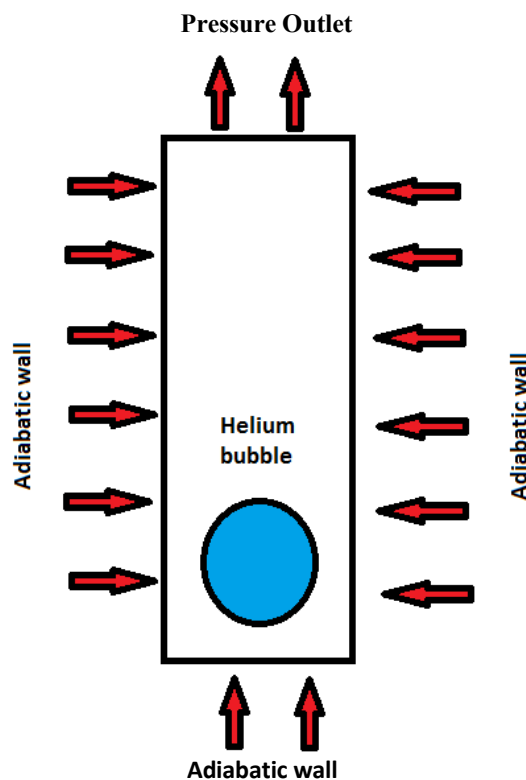


Figure 2.2: Pictorial representation of problem with all the boundary conditions

In this problem, numerical study of helium bubble movement in a rectangular tube is carried out. The rectangular tube is filled with water. The temperature of both helium bubble and water is kept as 300 K. Adiabatic wall condition is given. The problem has

been analyzed by considering two different diameters. The dimension of the rectangular domain is taken as  $30\text{ mm} \times 30\text{ mm} \times 1\text{ m}$ .

**2.2.1** The diameter of helium bubble is taken as 15 mm and is placed at  $X = 15\text{ mm}$  from bottom wall.

**2.2.2** The diameter of helium bubble is taken as 20 mm and is placed at  $X = 15\text{ mm}$  from bottom wall.

# **CHAPTER 3:**

# **NUMERICAL**

# **SIMULATION**

The advancement of high speed computing with high accuracy of numerical methods for the solution of physical problems, has made a big revolution in the way we approach fluid dynamics and heat transfer problems. Computational Fluid Dynamics (CFD) made the analysis of complex flow geometries much easier as compared to the earlier conventional methods. CFD provides a platform for combining fluid dynamics and numerical analysis. The need for CFD was realized in 1960s and 1970s and was specifically developed for aerospace industries. Now a days the use of CFD can be seen in every disciplines such as mechanical, civil, electronics, electrical, chemical, ocean science, and biomedical engineering etc. CFD reduces the total cost and total time required for the analysis of a problem by replacing the analytical studies and experimental testing with numerical simulation methods.

CFD software consists of following three basic elements:

1. Pre processor
2. Main Solver
3. Post processor

The first step in CFD modelling is to understand the actual problem and define the computational domain. After mesh is generated which is a critical step of the pre-processing activity. This is the most important step of CFD because the accuracy of the solution and the computational time for the problem both depends on the mesh structure. Finer the mesh, more accurate the solution is but the mesh should not made very fine as that would take more time for computation. So an optimize grid size should be selected.

The role of the solver is to solve the equations as per the given information. It is consider to be the brain of the CFD software. The post- processor is the last step of the

CFD software. It gives the useful data that can be used to get the final result and to draw some conclusion. The results that we get here may be in the form of vector plots of vector quantities such as velocity or in the form of contour plots of scalar quantities such as temperature, pressure etc.

### 3.1 GEOMETRY

As per the requirement of the problem, a rectangular parallelepiped domain has been created using workbench. Once the geometry is created with the given dimension, meshing operation is performed using workbench.

### 3.2 GRID PATTERN

Uniform grid size of 0.5 mm has been chosen after performing a lengthy verification for various grid sizes. It has been further verified in Fluent for the shape of the drop and bubble. Tetrahedron grid pattern has been selected to solve this multiphase flow problem.

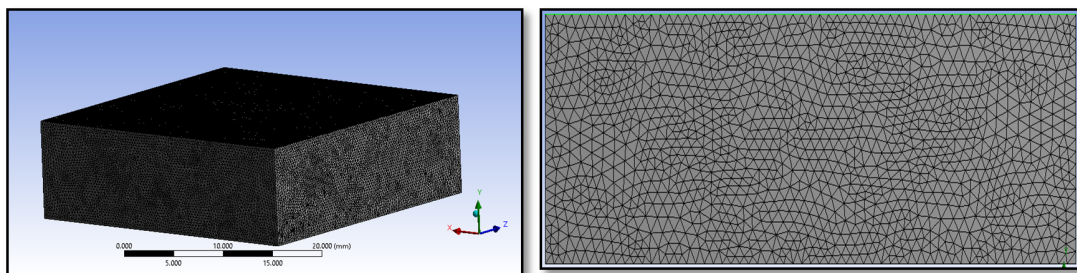


Figure 3.1: Figure showing the grid pattern used for this problem

### 3.3 SOLUTION STRATEGY

The modeling and meshing of all the problems have been done using workbench and hydrodynamic study has been carried out with the help of fluent. VOF model has been used for the analysis of the problem. Pressure-based 3-D solver has been used to numerically analyze the transient, non-linear, complex problems. The complex interface has been simulated using the Pressure Implicit Solution by Split Operator (PISO). Turbulent modeling, k- $\epsilon$  model is taken. Energy equation is switched off for all the cases of the 2.1 except 2.3.3 while it is switched ON for both the cases of 2.2.

### 3.4 GOVERNING EQUATIONS

The governing equations used for the analysis of this multi-phase flow problem are given below:

**The Continuity Equation:** principal of conservation of mass is used for the derivation of continuity equation and is an important equation in any CFD problem. This equation plays a crucial role in the stability of the problem. The continuity equation in vector form is given as:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{v} = 0 \quad (4.1)$$

**The Navier-Stoke's Equation (Momentum equations):** single set of momentum equations has been solved in VOF modelling with pressure-based solver. The equation is given as below:

$$\frac{\partial}{\partial t}(\rho \vec{v}) + \nabla \cdot (\rho \vec{v} \vec{v}) = -\nabla p + \nabla \cdot [\mu(\nabla \vec{v} + \nabla \vec{v}^T)] + \rho \vec{g} + \vec{F} \quad (4.2)$$

**The Standard k-ε Model Equations:** standard k-ε model has been used for the turbulent modelling and is given below:

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(\rho k u_i) = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + G_k + G_b - \rho \varepsilon - Y_M + S_k \quad (4.3)$$

$$\begin{aligned} \frac{\partial}{\partial t}(\rho \varepsilon) + \frac{\partial}{\partial x_i}(\rho \varepsilon u_i) \\ = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + C_{1\varepsilon} \frac{\varepsilon}{k} (G_k + C_3 G_b) - C_{2\varepsilon} \rho \frac{\varepsilon^2}{k} + S_\varepsilon \end{aligned} \quad (4.4)$$

Where,  $\mu_t$  is called as turbulent viscous coefficient and is given as;

$$\mu_t = \rho C_\mu \frac{k^2}{\varepsilon} \quad (4.5)$$

### 3.5 BOUNDARY CONDITIONS

For 2.1, 2.1.1 the value of pressure inlet and pressure outlet is taken as atmospheric. The wall is given as adiabatic, no slip boundary condition. The magnitude of velocity inlet for 2.1.1.1, 2.1.1.2, and 2.1.1.3 is taken as 1.1 m/s, 3.2 m/s and 4.39 m/s respectively.

For 2.1.2, the value of pressure inlet and pressure outlet is kept as atmospheric. Adiabatic, no slip boundary condition is given to the wall. The magnitude of velocity inlet for all the subsets is taken as 3.2 m/s.

For case 2.1.3, pressure inlet and pressure outlet are assigned the value which is equal to the atmospheric pressure. In 2.1.3.1, wall is given the heat flux whose value is taken as 1 mw and the magnitude of velocity inlet is taken as 2.2 m/s. In 2.1.3.2, wall is given the heat flux whose value is taken as 10 mw and the magnitude of velocity inlet is



taken as 2.2 m/s. In 2.1.3.3, wall is given the heat flux whose value is taken as 100 mw and the magnitude of velocity inlet is taken as 2.2 m/s. For 2.2, the wall is given adiabatic, no slip condition and the value of pressure outlet is taken as atmospheric pressure.

### **3.6 RESIDUAL AND CONVERGENCE**

The accuracy level in RESIDUAL PLOT for continuity, X, Y, Z momentum equation, turbulent dissipation and turbulent kinetic energy equation is taken as  $10^{-6}$  and the accuracy level for energy equation is set to  $10^{-9}$ .

# **CHAPTER 4:**

# **RESULTS**

# **AND**

# **DISCUSSION**

The detail results of the problems as defined in chapter 2 are categorically presented here. The extensive discussion for each of the obtained result is also elaborately described in the present chapter.

## 4.1 DYNAMICS OF DROP WITH AND WITHOUT ADIABATIC WALL

Several cases have been studied under this section, which are given below.

**4.1.1 The effect of  $Re$ :** The effect of Reynolds Number on the dynamics of the drop on flat horizontal plate under shear flow with adiabatic condition by considering three different sub-cases with drop diameter equals to 6 mm, Drop-center location: (15, 0, 10). In this case, the effect of shear velocity on the motion and shape change phenomena of the drop placed on a horizontal solid flat plate is studied. The data is saved after every 100 time steps with each time step size of 0.0001. The study has been conducted by taking three different values of Reynolds number as 1500, 3000 and 4000.

**4.1.1.1  $Re = 1500$ :** The phase contours showing the shape change of the drop at various time instants during its motion for the Reynolds number equal to 1500.

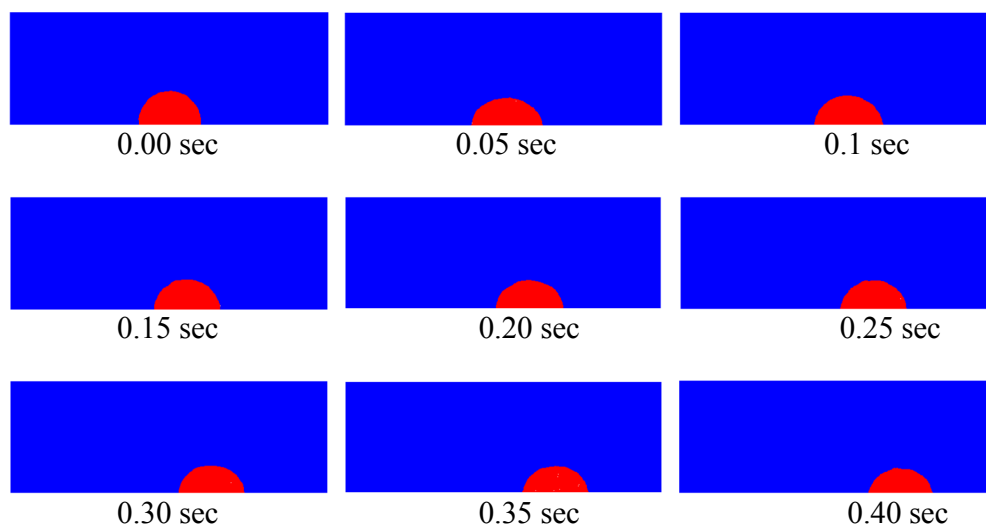


Figure 4.1: Phase contours at various time instant on vertical mid-plane ( $Re = 1500$ , Drop diameter = 6 mm, Drop-center location:  $X=15$  mm,  $Y= 0$  mm,  $Z=10$  mm)

**4.1.1.2  $Re = 3000$ :** The phase contours showing the shape change of the drop at various time instants during its motion for the Reynolds number equal to 3000.

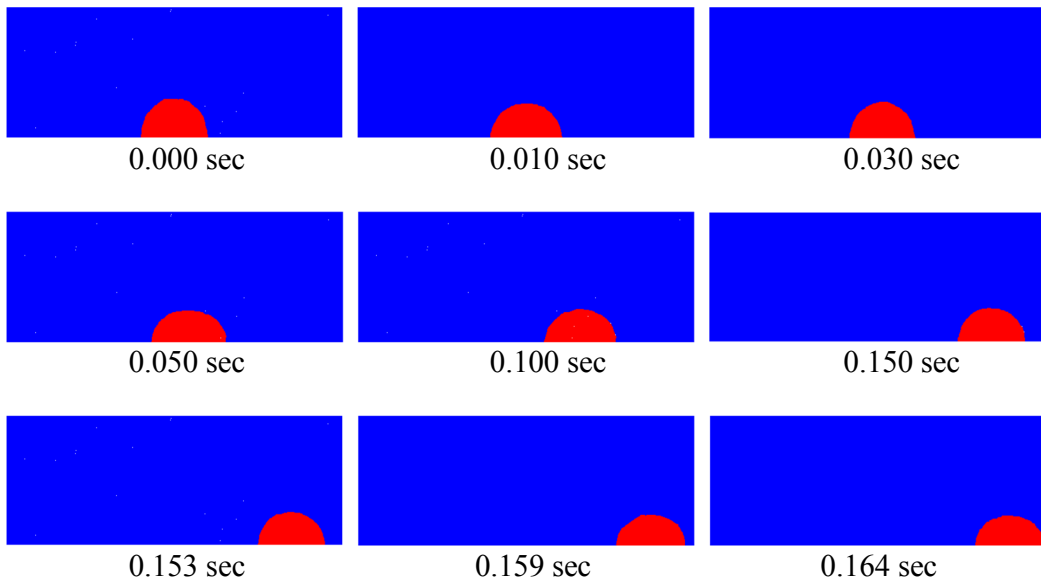


Figure 4.2: Phase contours at various time instant on vertical mid-plane ( $Re = 3000$ , Drop diameter = 6 mm, Drop-center location:  $X=15$  mm,  $Y=0$  mm,  $Z=10$  mm)

**4.1.1.3  $Re = 4000$ :** The phase contours showing the shape change of the drop at various time instants during its motion for the Reynolds number equal to 4000.

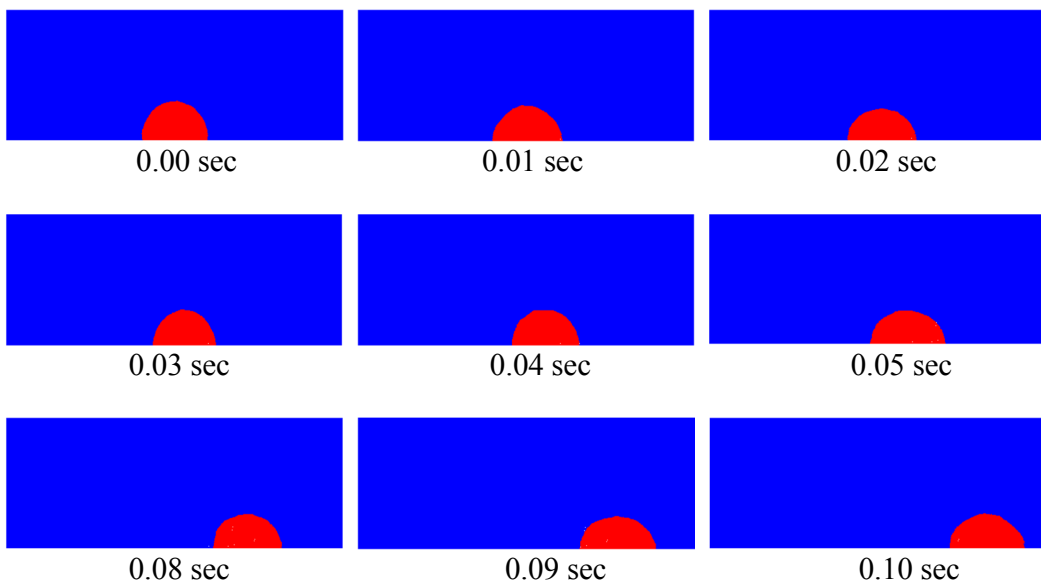


Figure 4.3: Phase contours at various time instant on vertical mid-plane ( $Re = 4000$ , Drop diameter = 6 mm, Drop-center location:  $X=15$  mm,  $Y=0$  mm,  $Z=10$  mm)

Velocity has been calculated at various time-instant during the motion of the drop on the solid horizontal surface. The velocity thus found is then plotted against the time interval and the graph thus obtained is shown below.

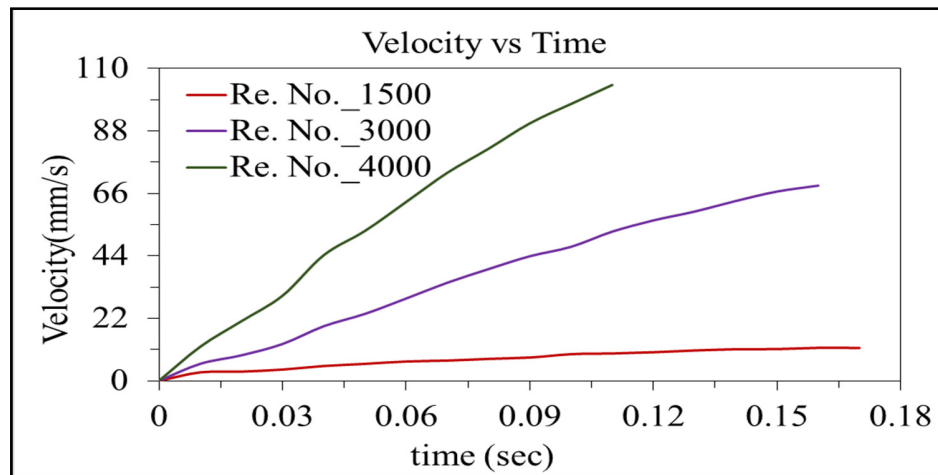


Figure 4.4: Variation of Drop velocity with time for different Reynolds number (Drop diameter: 6 mm, Drop-center location: X=15 mm, Y= 0 mm, Z=10 mm)

Drop shows the oscillating motion during its movement on the horizontal solid surface and this behavior is not much affected by varying the Reynolds No. This behavior of the drop is due to the two forces acting on the drop i.e. gravity force and the shear force. It has also been observed that as the Reynolds No. increases, the slope of the velocity-time graph also increases. At higher Reynolds Number, the drop moves with higher velocity as compare to low Reynolds Number.

**4.1.2 The effect of drop location:** The effect of location on the dynamics of the drop with diameter 10mm, and Reynolds number 3000 on flat horizontal plate under shear flow with adiabatic condition by considering different sub-cases.

To study the effect of location of the drop, results of sub-cases of case 2.1.2 are analyzed. The data has been saved after every 50 time steps with time step size of 0.0001.

**4.1.2.1 Drop-Center Location (5, 0, 10) with D = 6 mm:** Phase contours showing the shape change of the drop at various time instants during its motion for the Drop diameter 6 mm, Drop-center location  $X=5$  mm,  $Y=0$  mm,  $Z=10$  mm and value of Reynolds number is taken as 3000.

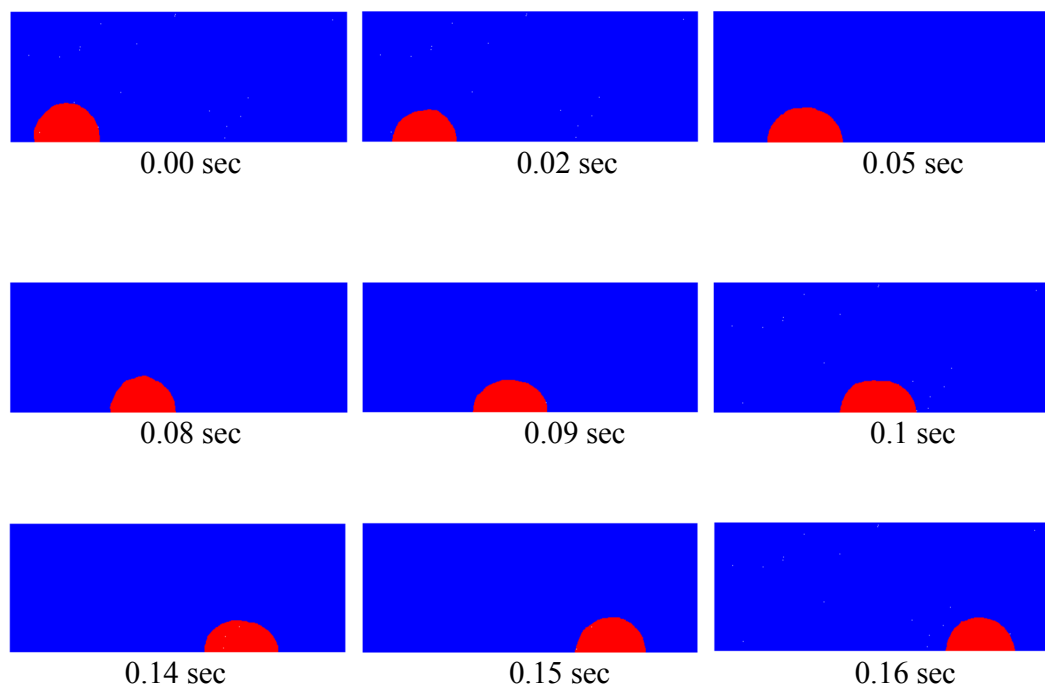


Figure 4.5: Phase contours at various time instant on vertical mid-plane (Reynolds No.: 3000, Drop diameter: 6 mm, Drop-center location:  $X=5$  mm,  $Y=0$  mm,  $Z=10$  mm)

**4.1.2.2 Drop-Center Location (15, 0, 10) with D = 6 mm:** Phase contours showing the shape change of the drop at various time instants during its motion for the Drop diameter 6 mm, Drop-center location  $X=15$  mm,  $Y=0$  mm,  $Z=10$  mm and value of Reynolds number is taken as 3000.

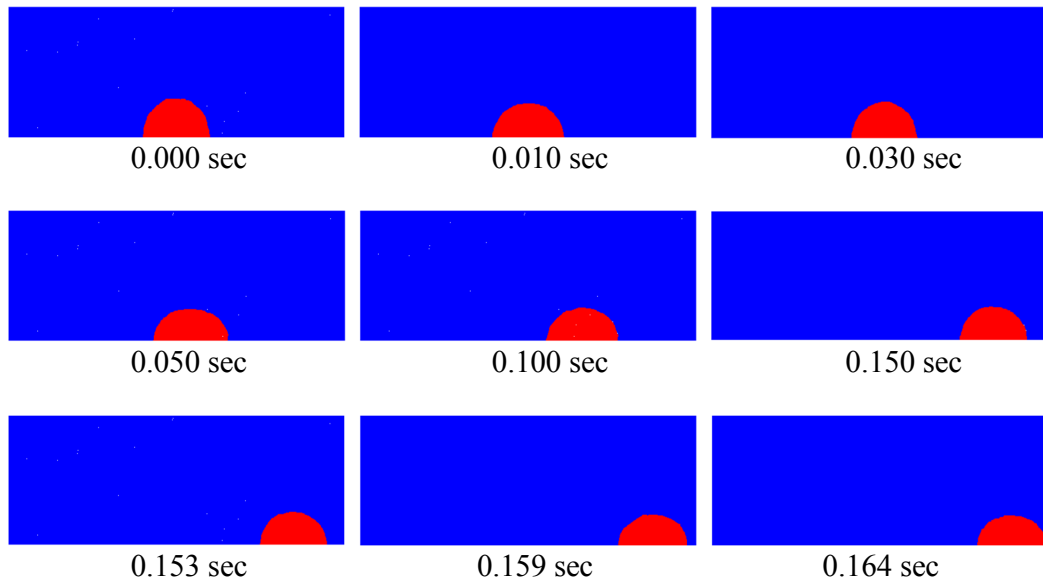


Figure 4.6: Phase contours at various time instant on vertical mid-plane ( $Re = 3000$ , Drop diameter = 6 mm, Drop-center location:  $X=15$  mm,  $Y=0$  mm,  $Z=10$  mm)

**4.1.2.3 Drop-Center Location (5, 0, 10) with  $D = 10$  mm:** Phase contours showing the shape change of the drop at various time instants during its motion for the Drop diameter 10 mm, Drop-center location  $X=15$  mm,  $Y=0$  mm,  $Z=10$  mm and value of Reynolds number is taken as 3000.

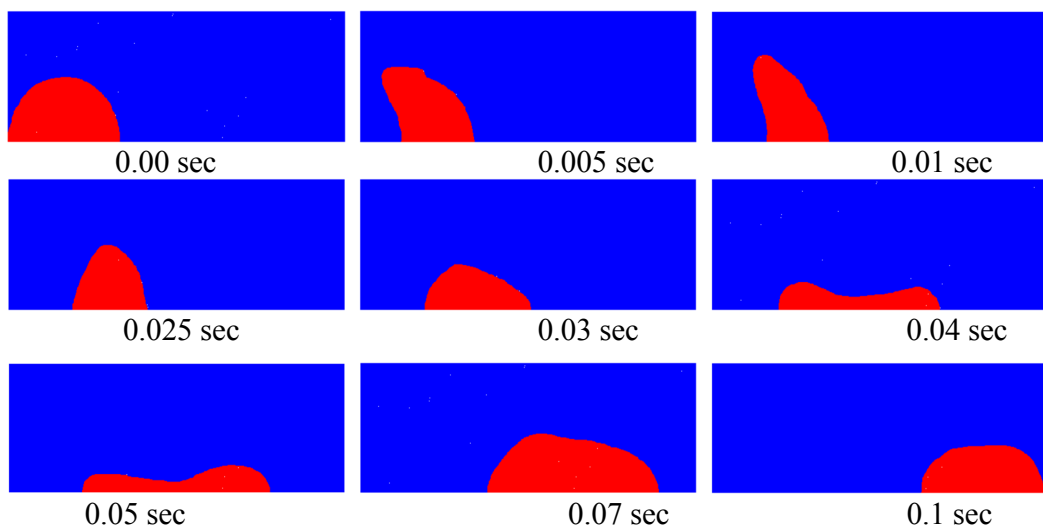


Figure 4.7: Phase contours at various time instant on vertical mid-plane (Reynolds No.: 3000, Drop diameter: 10 mm, Drop-center location:  $X=5$  mm,  $Y=0$  mm,  $Z=10$  mm).

**4.1.2.4 Drop-Center Location (15, 0, 10) with D = 10 mm:** Phase contours showing the shape change of the drop at various time instants during its motion for the Drop diameter 10 mm, Drop-center location  $X=15$  mm,  $Y=0$  mm,  $Z=10$  mm and value of Reynolds number is taken as 3000.

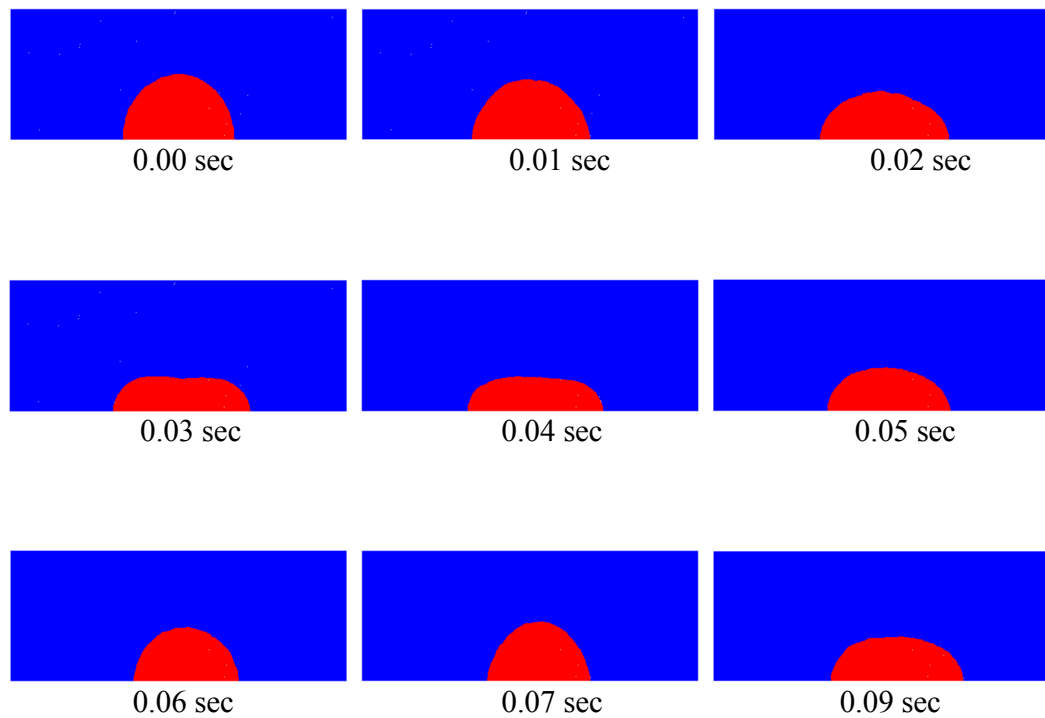


Figure 4.8: Phase contours at various time instant on vertical mid-plane (Reynolds No.: 3000, Drop diameter: 10 mm, Drop-center location:  $X=15$  mm,  $Y=0$  mm,  $Z=10$  mm).

The velocity of the drop is calculated at various time instants to analyze the effect of location of the drop on the velocity acquired by the drop at various time instants.



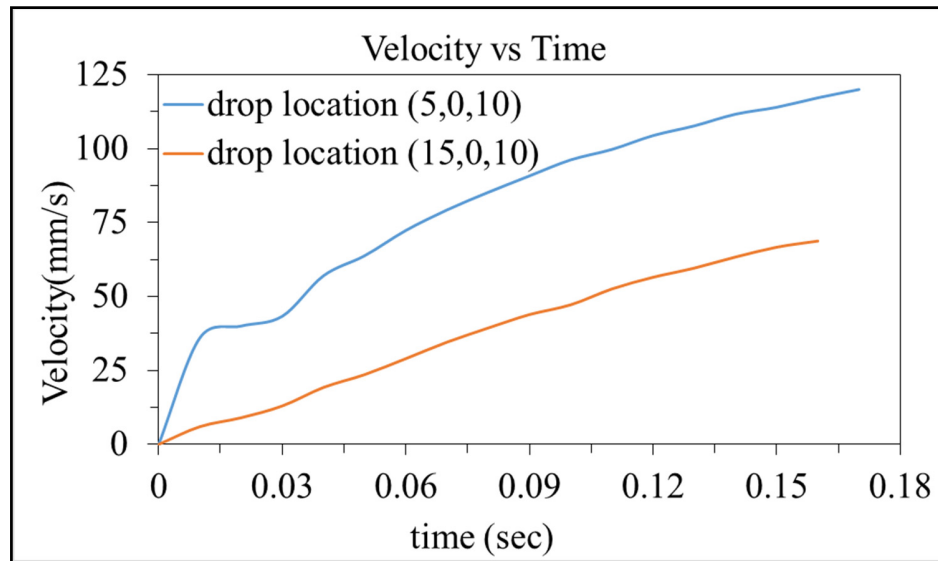


Figure 4.9: Variation of Drop velocity with time for different location of the Drop-center (Drop diameter: 6 mm, Reynolds no. 3000)

According to this graph, the velocity acquired by the drop when placed close to the velocity inlet plane is more compare to the velocity acquired by the drop when placed away from the velocity inlet plane. Apart from this, the graph also shows that more fluctuations are observed initially and after some time, the velocity-time plot becomes more uniform.

**4.1.3 The effect of size:** The effect of size on the dynamics of the drop on flat horizontal plate under shear flow with adiabatic condition by considering different sub-cases.

**4.1.3.1 D = 6 mm located at (15, 0, 10):** Phase contours showing the shape change of the drop at various time instants during its motion for the Drop diameter 6 mm, Drop-center location X= 15 mm, Y= 0 mm, Z= 10 mm and value of Reynolds number is taken as 3000.

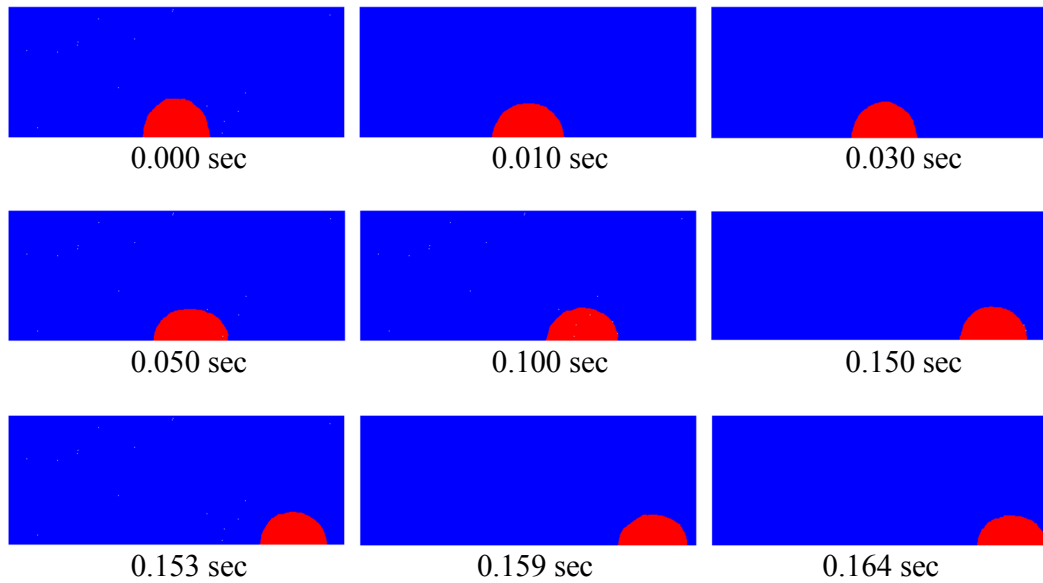


Figure 4.10: Phase contours at various time instant on vertical mid-plane ( $Re = 3000$ , Drop diameter = 6 mm, Drop-center location:  $X=15$  mm,  $Y=0$  mm,  $Z=10$  mm)

**4.1.3.2  $D = 10$  mm located at (15, 0, 10):** Phase contours showing the shape change of the drop at various time instants during its motion for the Drop diameter 6 mm, Drop-center location  $X=15$  mm,  $Y=0$  mm,  $Z=10$  mm and value of Reynolds number is taken as 3000.

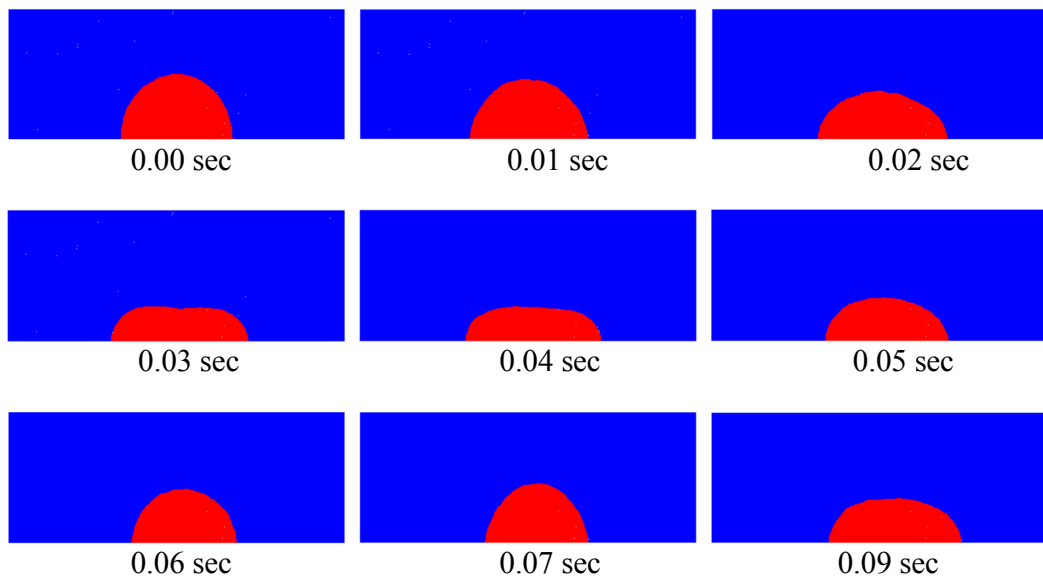


Figure 4.11: Phase contours at various time instant on vertical mid-plane (Reynolds No.: 3000, Drop diameter: 10 mm, Drop-center location:  $X=15$  mm,  $Y=0$  mm,  $Z=10$  mm).

**4.1.3.3 D = 6 mm located at (5, 0, 10):** Phase contours showing the shape change of the drop at various time instants during its motion for the Drop diameter 6 mm, Drop-center location  $X=5$  mm,  $Y=0$  mm,  $Z=10$  mm and value of Reynolds number is taken as 3000.

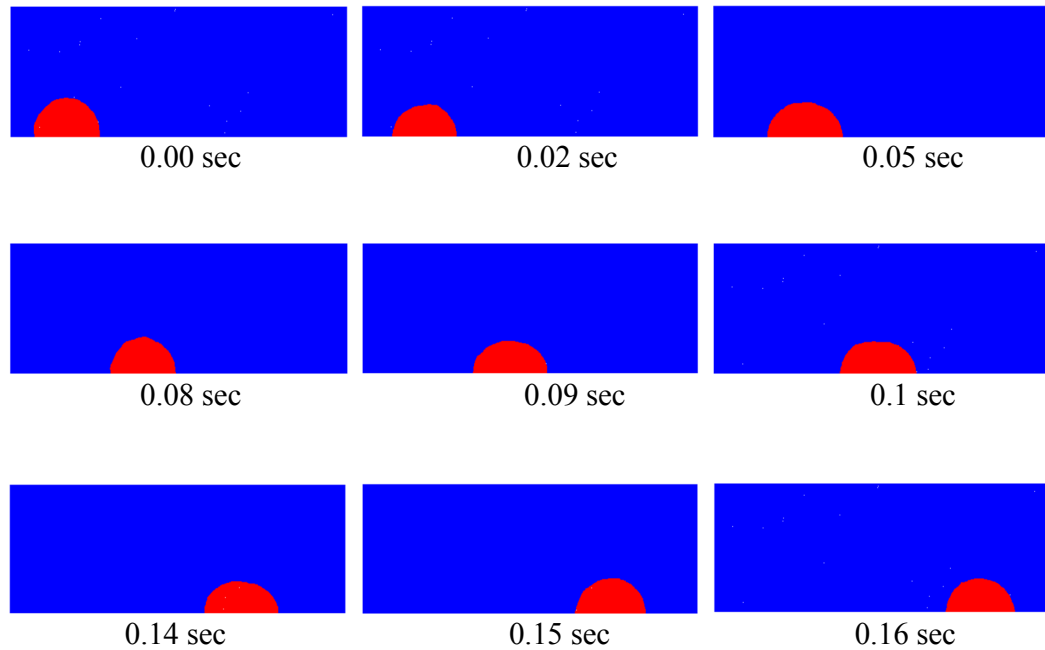


Figure 4.12: Phase contours at various time instant on vertical mid-plane (Reynolds No.: 3000, Drop diameter: 6 mm, Drop-center location:  $X=5$  mm,  $Y=0$  mm,  $Z=10$  mm)

**4.1.3.4 D = 10 mm located at (5, 0, 10):** Phase contours showing the shape change of the drop at various time instants during its motion for the Drop diameter 10 mm, Drop-center location  $X=5$  mm,  $Y=0$  mm,  $Z=10$  mm and value of Reynolds number is taken as 3000.

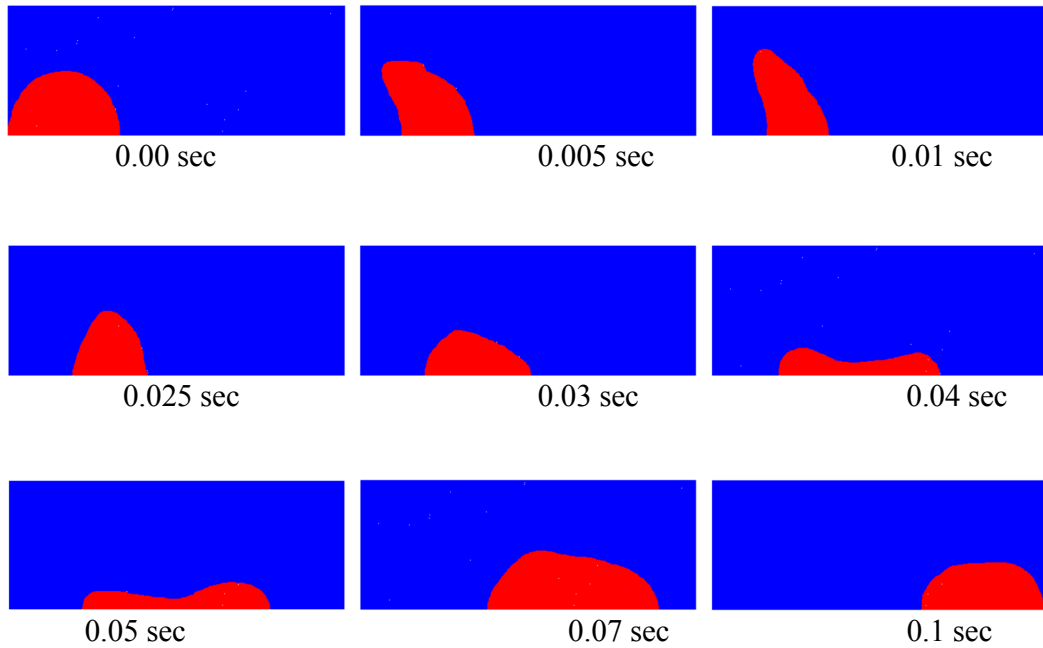


Figure 4.13: Phase contours at various time instant on vertical mid-plane (Reynolds No.: 3000, Drop diameter: 10 mm, Drop-center location: X=15 mm, Y= 0 mm, Z=10 mm).

To analyze the effect of size of the drop on the velocity acquired by the drop during its motion, the velocity of the drop at various time instant is calculated and is plotted as shown below.

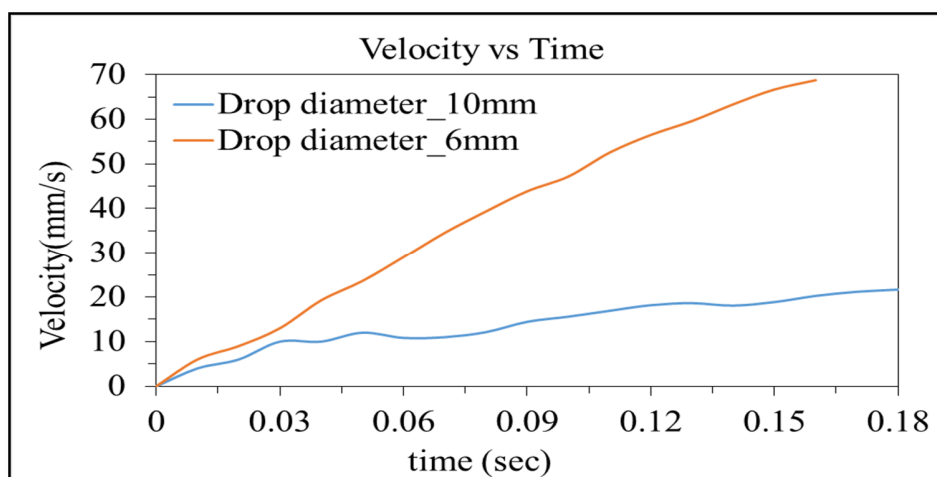


Figure 4.14: Variation of Drop velocity with time for different Drop diameter (Drop-center location: X=15 mm, Y= 0 mm, Z=10 mm and Reynold No. : 3000).

From the graph, it has been clear that drop size affects the velocity acquired by the drop. The drop with large diameter takes more time to cover the same distance as taken by the smaller diameter drop. The velocity acquired by the drop with diameter 10mm is less compared to the drop with diameter 6 mm as shown by the graph. As the drop size is increased, the shape change of the drop during its motion is more clearly visible. When larger size drop is placed close to inlet plane, the drop shows sudden change in shape initially and after some time follows the normal oscillating movement

**4.1.4 The effect of heat flux:** The effect of heat flux on the dynamics of the drop on flat horizontal plate under shear flow with non-adiabatic condition by considering three different sub-cases with drop diameter equals to 6 mm and drop-centre location equals to (15, 0, 10) and Reynolds number equals to 2000

The volume fraction of drop at various time instant is taken for the all three value of heat flux and is plotted against the time instants to study the effect of heat flux on the drop while it moves on the horizontal solid surface.

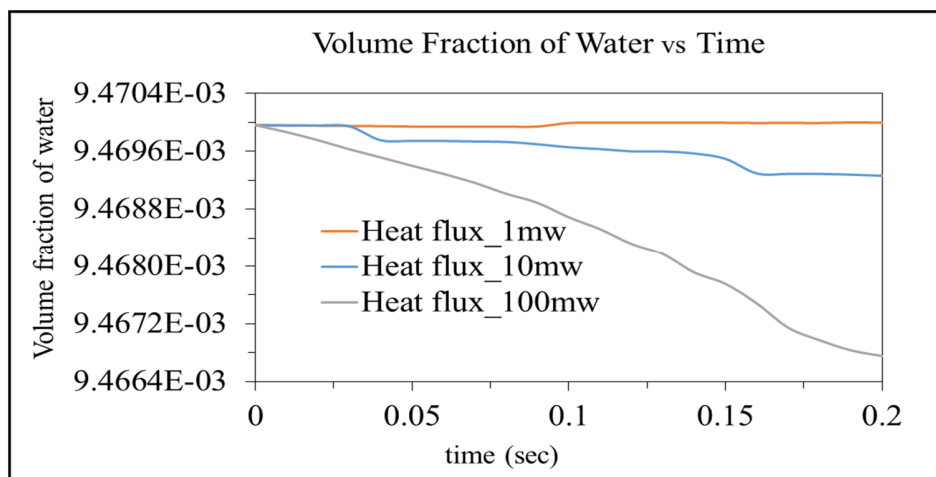


Figure 4.15: Variation of Drop velocity with time for different value of heat flux at the solid surface (Drop-center location: X=5 mm, Y= 0 mm, Z=10 mm, Drop diameter: 6 mm, and Reynolds no. 2000.

When the heat flux boundary conditions (non-adiabatic) on the solid horizontal surface is used, the size ( volume) of the drop will reduces as the evaporation will take part in the phenomena.

## 4.2 DYNAMICS OF HELIUM BUBBLE IN A RECTANGULAR CHANNEL

**4.2.1 Bubble diameter = 15 mm:** In this case the dynamics of helium bubble with diameter 15 mm is studied while it is moving through the vertical channel which is filled with water. Phase contours of helium bubble at various time steps are plotted on the vertical mid-plane.

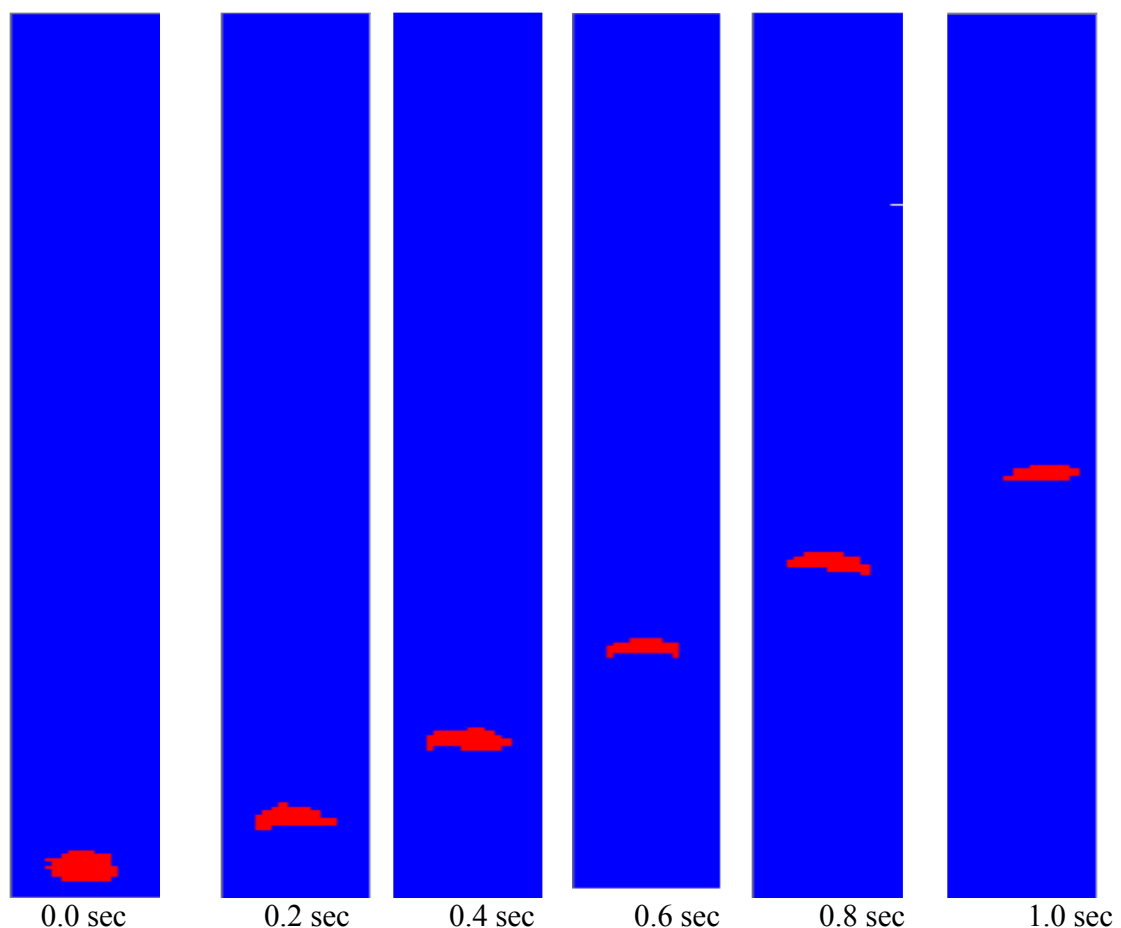


Figure 4.16: Phase contours for helium bubble with dia. 15 mm at various time instant on vertical mid-plane.

**4.2.2 Bubble diameter = 20 mm:** In this case the dynamics of helium bubble with diameter 20 mm is studied while it is moving through the vertical channel which is filled with water.

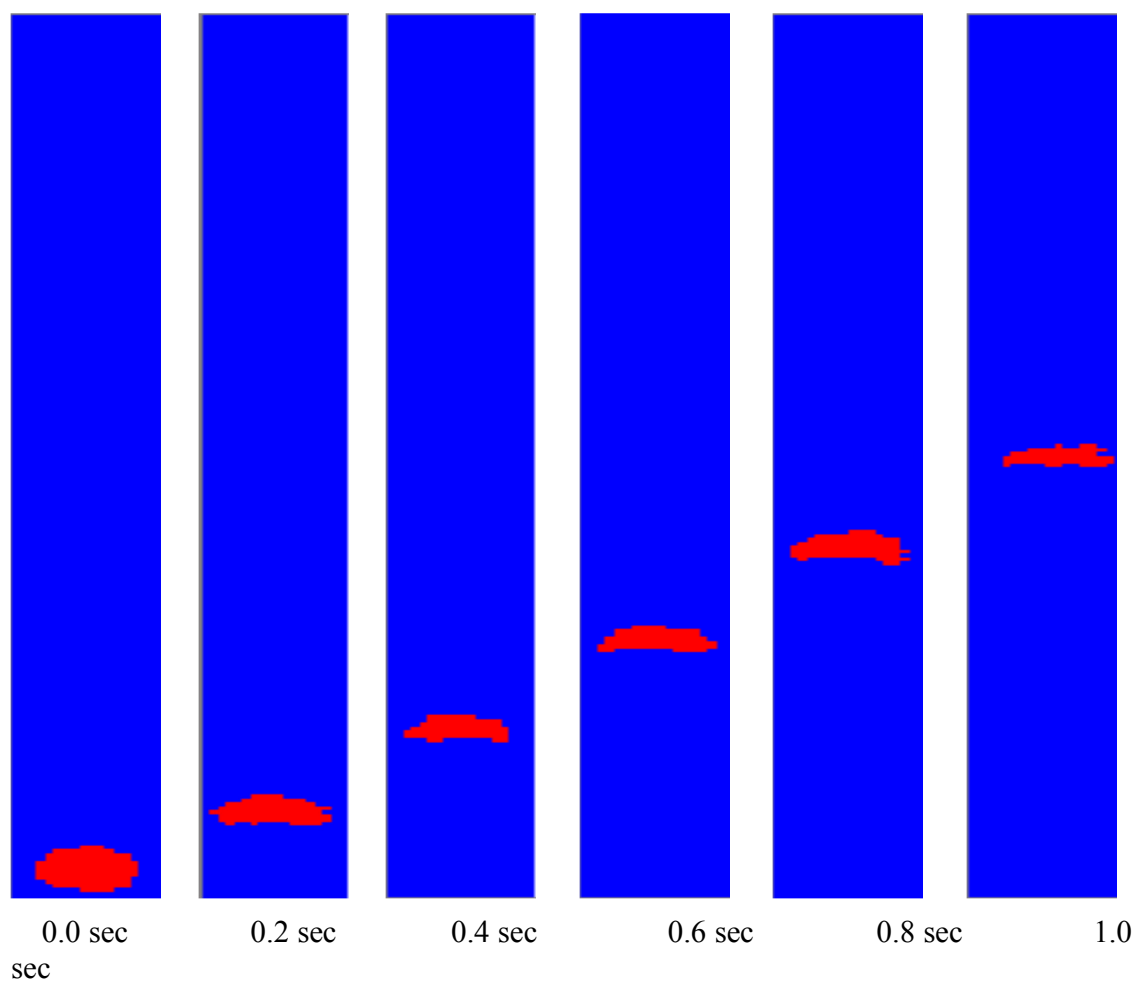


Figure 4.17: Phase contours for helium bubble with dia. 20 mm at various time instant on vertical mid-plane.

The velocity of the bubble at various time instants has been calculated and plotted against the time instants.

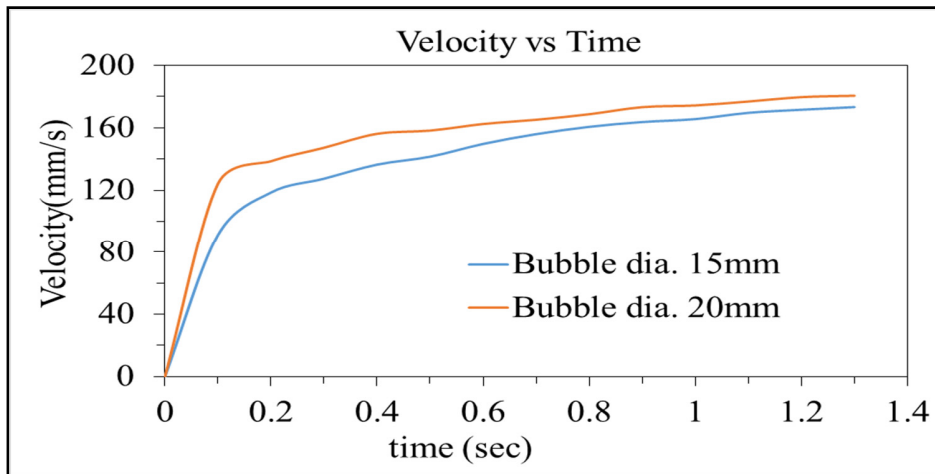


Figure 4.18: the graph showing the velocity of the bubble with diameter 15 mm and velocity of the bubble with diameter 20 mm plotted against the time instant

There is a sudden rise in the velocity of the bubble for both the diameters till 0.1 second. After 0.1 second, the rise in velocity with time is very less as shown by the graph. The velocity acquired by larger diameter bubble is more compared to the velocity acquired by the smaller diameter bubble. This may be because with the increase in diameter of the bubble, the rise in buoyancy force is more compared to the rise in weight of the bubble.



# **CHAPTER 5:**

# **CONCLUSION**

# **AND**

# **FUTURE WORK**

## 5.1 CONCLUSION

The hydrodynamics of drop and bubble under various boundary conditions (including adiabatic and non-adiabatic boundary condition) using numerical simulations have been studied. The effect of various parameters like: drop diameter, Reynolds no., position of drop on the dynamics is investigated. The numerical analysis of the drop and bubble are done using FVM with VOF model. At first, the kinetics of a drop on a horizontal solid surface under air flow (considering no heat transfer) is studied. Then the same problem with heat transfer through solid surface is considered. At last the hydrodynamics of a helium bubble through a vertical channel is studied. For each case the Reynolds number is varied for a considerable range.

1. It is clear from the present study that the drop motion on the horizontal flat surface is strongly affected by the Reynolds number. The time taken by the drop to cover the distance reduces as the Reynolds number increases.
2. Shape change phenomena of the drop is not much effected by the Reynolds number.
3. Position of the drop from the inlet has prominent effect on the drop dynamics. The effect of shear flow reduces as the distance of the drop from the inlet plane increases.
4. Size of the drop affects the velocity acquired by the drop at various time instant. Larger the size of the drop, lower the velocity acquired by the drop.
5. When the heat flux boundary conditions (non-adiabatic) on the solid horizontal surface is used, the size ( volume) of the drop will reduces as the evaporation will take part in the phenomena.

6. Size of the Helium bubble affects the dynamics of the bubble flowing through a vertical channel filled with water. On increasing the size of the bubble, it takes less time to cover the distance as the buoyancy force increases.

## **5.2 FUTURE WORK**

Effect of some more parameters such as contact angle, surface tension etc. needs to be analyzed. Evaporation phenomena of the drop on the flat plate under shear flow needs more investigation. Effect of other liquids on the dynamics of Helium bubble needs to be analyzed.

## REFERENCES

- Attinger, D., Zhao, Z., Poulikakos, D., 2000. An experimental study of molten microdroplet surface deposition and solidification: transient behavior and wetting angle dynamics. *Journal of Heat Transfer* 122, 544-556.
- Daniel, S., Chaudhury, M. K., 2002. Rectified motion of liquid drops on gradient surfaces induced by vibration. *Langmuir* 18, 3404-3407.
- Das, A. K., Das, P. K., 2009. Simulation of drop movement over an inclined surface using smoothed particle hydrodynamics. *Langmuir* 25, 11459-11466.
- Grissom, W. M., Wierum, F. A., 1981. Liquid spray cooling of a heated surface. *International Journal of Heat and Mass Transfer* 24, 261-271.
- Gunjal, P. R., Ranade, V. V., Chaudhari, R. V., 2005. Dynamics of drop impact on solid surface: experiments and VOF simulations. *AIChE Journal* 51, 59-78.
- Harlow, F. H., Shannon, J. P., 1967. The splash of a liquid drop. *Journal of Applied Physics* 38, 3855-3866.
- Javadi, A., Bastani, D., Krägel, J., Miller, R., 2009. Interfacial instability of growing drop: experimental study and conceptual analysis. *Colloids and Surfaces A: Physicochemical and Engineering Aspects* 347, 167-174.
- Loth, E., 2008. Quasi-steady shape and drag of deformable bubbles and drops. *International Journal of Multiphase Flow* 34, 523-546.
- Martinez, M. J., Udell, K. S., 1990. Axisymmetric creeping motion of drops through circular tubes. *Journal of Fluid Mechanics* 210, 565-591.
- Olbricht, W. L., Kung, D. M., 1992. The deformation and breakup of liquid drops in low Reynolds number flow through a capillary. *Physics of Fluids A: Fluid Dynamics* 4, 1347-1354.

- Pasandideh-Fard, M., Chandra, S., Mostaghimi, J., 2002. A three-dimensional model of droplet impact and solidification. *International Journal of Heat and Mass Transfer* 45, 2229-2242.
- Pismen, L. M., Thiele, U., 2006. Asymptotic theory for a moving droplet driven by a wettability gradient. *Physics of Fluids* 18, 042104.
- Šikalo, Š., Wilhelm, H. D., Roisman, I. V., Jakirlić, S., Tropea, C., 2005. Dynamic contact angle of spreading droplets: Experiments and simulations. *Physics of Fluids* 17, 062103.
- Staben, M. E., Davis, R. H., 2005. Particle transport in Poiseuille flow in narrow channels. *International Journal of Multiphase Flow* 31, 529-547.
- Subramanian, R. S., Moumen, N., McLaughlin, J. B., 2005. Motion of a drop on a solid surface due to a wettability gradient. *Langmuir* 21, 11844-11849.
- Trapaga, G., & Szekely, J., 1991. Mathematical modeling of the isothermal impingement of liquid droplets in spraying processes. *Metallurgical Transactions B* 22, 901-914.
- Tsai, T. M., Miksis, M. J., 1994. Dynamics of a drop in a constricted capillary tube. *Journal of Fluid Mechanics* 274, 197-217.